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Global Stabilization of Large-Scale Hydraulic Networks Using Quantized Proportional Control^{*}

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Abstract: An industrial case study involving a hydraulic network underlying a district heating system is investigated. The flexible structure of the network calls for control structure which is able to handle changes in the network structure. For this purpose a set of decentralized proportional controllers have been proposed. These controllers make use only of locally available information, and in order to make implementation of the control laws possible, the control signals are required to be communicated across the network. To accommodate this a quantized version of the control laws are considered, and the results show that the designed closed loop system maintains its stability properties despite the structural changes introduced in the system.

Keywords: nonlinear systems, robust control, decentralized control

1. INTRODUCTION

The work presented here considers the investigation of an industrial case study. The case study involves a large-scale hydraulic network which underlies a district heating system. Specifically, the case study regards a new paradigm for the design of district heating systems. By reducing the diameters of the pipes in the network the heat dispersion can be reduced, making it possible to reduce the heat losses in the system by 20 % to 50 % (Kallesøe, 2007). Furthermore, the new paradigm allows for a more flexible network structure, which calls for a new control structure which is able to handle structural changes in the network, such as the addition or removal of end-users (Kallesøe, 2007). The case study is part of the ongoing research program *Plug & Play Process Control* (Stoustrup, 2009), which considers automatic reconfiguration of the control system whenever components such as actuators or sensors are added to or removed from the system. The case study has been proposed by one of the industrial partners involved in the program.

A set of decentralized proportional control actions are proposed to meet the control objective in the system, which is to maintain the pressure across the so-called end-user valves at a piecewise constant reference point. The controllers use only locally available information, which is the pressure measurement at each end-user.

Reducing the pipe diameter in the district heating system, has the consequence that the pressure losses across the pipes are increased. This is compensated by distributing a number of (boosting) pumps across the network in order to meet pressure constraints (Kallesøe, 2007). This means that the actuators are geographically separated

from the controllers, making it necessary to communicate the control signals over a communication network. In order to accomplish this, the control signals are quantized in the sense that they are piecewise constant taking value in a finite set. This makes it possible to send them across a finite bandwidth network.

The result presented here shows that, given a properly designed quantizer, the closed loop system with the quantized control actions is globally attracted to a compact set, which can be made arbitrarily small by a proper design of the controller gains and quantization parameters. Furthermore, since the result is independent of the number of end-users in the system, the closed loop system will maintain these stability properties whenever end-users are added to or removed from the system.

The model of the system is introduced in Section 2. In Section 3, the control objective is introduced along with the proposed controllers and the quantization map. In Section 4, the stability properties of the closed loop system are analysed. Section 5 presents the result of numerical simulations performed on the closed loop system. Finally, conclusions are drawn in Section 6.

1.1 Preliminaries

- Throughout the following, C^1 denotes the set of continuously differentiable functions.
- A continuous function (map) is said to be proper if the inverse image of a compact set is compact.
- A function $f : \mathbb{R} \rightarrow \mathbb{R}$ is called monotonically increasing if it is natural order preserving, *i.e.*, for all x and y such that $x \leq y$ then $f(x) \leq f(y)$.
- $M(n, m; \mathbb{R})$ denotes the set of $n \times m$ matrices with real entries and $M(n; \mathbb{R}) = M(n, n; \mathbb{R})$.
- $A > 0$ means that A is a positive definite matrix, *i.e.*, $A = A^\top$ and $x^\top A x > 0, \forall x \neq 0$.

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- $A = \text{diag}(x_i)$ means that A has entries x_i on the main diagonal and zero elsewhere.
- For two vectors $a, b \in \mathbb{R}^n$, $\langle a, b \rangle$ denotes the Euclidean scalar product.
- $B_r(x) = \{y \in \mathbb{R}^n \mid |y - x| < r\}$.

2. SYSTEM MODEL

In this section, the model of the large-scale hydraulic network will be recalled. The model is fully described in (DePersis and Kallesøe, 2009).

2.1 Component Models

The hydraulic network is comprised of three types of two-terminal components: valves, pipes and pumps as well as a number of interconnections between these components. These components are characterized by dual variables, the first of which is the pressure drop Δh across them

$$\Delta h = h_i - h_j, \quad (1)$$

where i, j are nodes of the network; h_i, h_j are the relative pressures at the nodes.

The other variable characterizing the components is the fluid flow q through them. The components in the network are governed by dynamic or algebraic equations describing the relation between the two dual variables.

Valves A valve in the hydraulic network is described by the following algebraic relation

$$h_i - h_j = \mu(k_v, q), \quad (2)$$

where k_v is the hydraulic resistance of the valve; $\mu(k_v, \cdot) \in C^1$ is proper and for any constant value of k_v is zero at $q = 0$ and monotonically increasing.

Pipes A pipe is described by the dynamic equation

$$\mathcal{J}\dot{q} = (h_i - h_j) - \lambda(k_p, q) \quad (3)$$

where \mathcal{J} and k_p are parameters of the pipe; $\lambda(k_p, \cdot) \in C^1$ have the same properties as $\mu(k_v, \cdot)$.

Pumps A (centrifugal) pump is a component which is able to maintain a desired pressure difference Δh across it regardless of the value of the fluid flow through it. This means that the constitutive law of the pump is

$$h_i - h_j = -\Delta h_p \quad (4)$$

where Δh_p is a signal, which for the purpose of the present exposition, is viewed as a control input.

Typically, exact values of the parameters k_v and k_p are not known but will be assumed to be positive and to take values in a known compact set. Furthermore, the functions $\mu(k_v, \cdot)$ and $\lambda(k_p, \cdot)$ are not precisely known. Only their properties of being in C^1 , proper, monotonic increasing and zero for $q = 0$ will be guaranteed.

The varying demand for heating at the end-users in the hydraulic network is modelled by a (end-user) valve for which the hydraulic resistance can be changed in a piecewise constant way. Thus, a distinction is to be made between the end-user valves and the remaining valves in the network. Likewise, a distinction is made between end-user pumps and booster pumps in the network. The later are pumps placed in the network to meet constraints on

the relative pressures across the network. The former are pumps located in the vicinity of the end-user valves and are mainly used to meet the demands of the end-users.

2.2 Network Model

The model of the hydraulic network has been derived by using tools from circuit theory (DePersis and Kallesøe, 2009). The network is comprised of m components and n end-users, where $m > n$. To the network is associated a graph \mathcal{G} , where the nodes of \mathcal{G} coincides with the terminals of the components and the edges of \mathcal{G} coincides with the components themselves. A vector of independent flow variables is identified as the flows through the chords of \mathcal{G} . These flow variables have the property that they can be set independently of all other flow variables in the network. To each chord in \mathcal{G} (i.e. to each independent flow variable) a fundamental flow loop is associated. Along each of the fundamental flow loops Kirchhoff's voltage law holds, which can be expressed as

$$B\Delta h = 0, \quad (5)$$

where $B \in M(n, m; \mathbb{R})$ is called the fundamental loop matrix; Δh is a vector consisting of the pressure drops across the components in the network. The fundamental loop matrix B consists of $-1, 0, 1$, depending on the structure of the network.

The class of hydraulic networks which are considered here satisfy the following two assumptions:

Assumption 1. (DePersis and Kallesøe, 2009) Each end-user valve is in series with a pipe and a pump, as seen in Fig. 1. Furthermore, each chord in \mathcal{G} corresponds to a pipe in series with a user valve.

Assumption 2. (DePersis and Kallesøe, 2009) There exists one and only one component called the heat source. It corresponds to a valve¹ of the network, and it lies in all the fundamental loops.

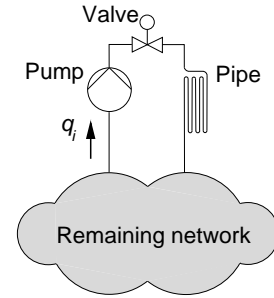


Fig. 1. The series connection associated with each end-user.

Proposition 3. (DePersis and Kallesøe, 2009) Any hydraulic network satisfying Assumption 1 admits the representation:

$$\mathcal{J}\dot{q} = f(K_p, K_v, B^\top q) + u \quad (6)$$

$$y_i(q_i) = \mu_i(k_{vi}, q_i), \quad i = 1, 2, \dots, n \quad (7)$$

where $q \in \mathbb{R}^n$ is the vector of independent flows; $u \in \mathbb{R}^n$ is a vector of independent inputs, which is a linear combination of the delivered pump pressures; y_i is the

¹ The valve models the pressure losses in the secondary side of the heat exchanger of the heat source.

measured pressure drop across the i th end-user valve (see (2)); $J \in M(n; \mathbb{R})$ and $J > 0$; K_p, K_v are vectors of system parameters; $f(K_p, K_v, B^\top q) \in C^1$; $\mu_i(k_{vi}, q_i)$ is the constitutive law of the i th end-user valve. In (7), it is assumed that the first n components coincide with the end-user valves.

Under Assumption 1 and Assumption 2, it is possible to select the orientation of the components in the network such that the entries of the fundamental loop matrix B are equal to 1 or 0.

A sketch of a simple district heating system with a heat source and two apartment buildings is illustrated in Fig. 2. The corresponding hydraulic network is illustrated in Fig. 3. The two end-users are represented by the series connections $\{c_{12}, c_{13}, c_{14}\}$ and $\{c_5, c_6, c_7\}$. The heat source is represented by the valve $\{c_{10}\}$ which models the pressure losses in the secondary side of the heat exchanger of the heat source.

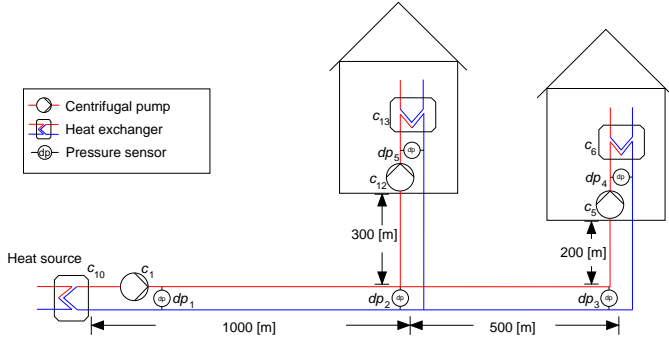


Fig. 2. A sketch of a small district heating system.

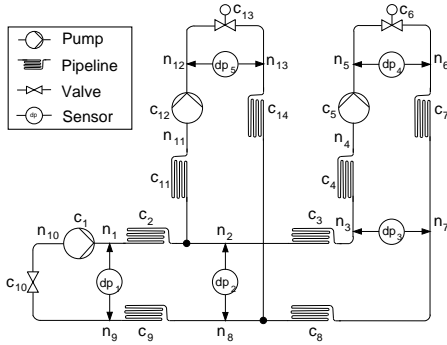


Fig. 3. The hydraulic network diagram.

3. PRESSURE REGULATION BY QUANTIZED CONTROL ACTIONS

This section introduces the control objective for the system in question along with a set of proposed control actions to accommodate this objective. Furthermore, a quantization map is introduced, which lets the control signals be piecewise constant taking values in a finite set.

3.1 Pressure Regulation Problem

It is desired to regulate the pressure (y_i) across the i th end-user valve to a given reference value (r_i) with the use of a feedback controller using locally available information

only. The vector $r = (r_1, \dots, r_n)$ of reference values take values in a known compact set \mathcal{R} :

$$\mathcal{R} = \{r \in \mathbb{R}^n \mid 0 < r_m \leq r_i \leq r_M\} \quad (8)$$

For the purpose of practical output regulation, a set of decentralized proportional controllers will be the focus of the work presented here. The controllers considered will be of the form:

$$u_i = -\gamma_i(y_i(q_i) - r_i), \quad i = 1, 2, \dots, n \quad (9)$$

where $\gamma_i > 0$ is the controller gain.

The controllers are decentralized in the sense that the individual controller use locally available information only. Thus, the control for the i th end-user uses information obtained only at the i th end-user, which is the measurement of the pressure across the end-user valve.

3.2 Quantization Map

This section describes the quantizers which will be used. To that end, let l be a positive integer, ψ_0 a positive real number, $\delta \in (0, 1)$, and $\psi_k = \rho^k \psi_0$ for $k = 1, 2, \dots, l$ with $\rho = \frac{1-\delta}{1+\delta}$ (i.e. $\psi_k = \frac{1-\delta}{1+\delta} \psi_{k-1}$). The following quantizer is then proposed (DePersis et al., 2010):

Let $\psi : \mathbb{R} \rightarrow \mathbb{R}$ be the map

$$\psi(u_i) = \begin{cases} \psi_0, & \frac{\psi_0}{1-\delta} < u_i \\ \psi_k, & \frac{\psi_k}{1+\delta} < u_i \leq \frac{\psi_k}{1-\delta}, \quad 0 \leq k \leq l \\ 0, & 0 \leq u_i \leq \frac{\psi_l}{1+\delta} \\ -\psi(-u_i), & u_i < 0 \end{cases} \quad (10)$$

The parameters l , ψ_0 and δ of the map (quantizer) are to be designed.

Define $\Psi : \mathbb{R}^n \rightarrow \mathbb{R}^n$ as $\Psi(u) = (\psi(u_1), \dots, \psi(u_n))^\top$, then the closed loop system with the quantized version of the proportional control actions is given as

$$J\dot{q} = f(K_p, K_v, B^\top q) + \Psi(u) \quad (11)$$

The piecewise constant map $\psi(\cdot)$ changes value whenever the continuous control signal u_i crosses some boundary, as defined in (10). The control signal u_i is governed by the expression (9), where r_i and γ_i are constant parameters. Thus, the quantized version ($\psi(u_i)$) of the control signal can be replaced with an expression depending on a quantized version of the system output ($\Upsilon(y_i)$) such that

$$\psi(-\gamma_i(y_i(q_i) - r_i)) = -\gamma_i(\Upsilon(y_i) - r_i). \quad (12)$$

To this end, the following quantized version of the output $y_i(q_i)$ is considered.

Define $\epsilon_i = y_i - r_i$ and let $\Upsilon : \mathbb{R} \rightarrow \mathbb{R}$ be the map

$$\Upsilon(y_i) = r_i + \begin{cases} \frac{\psi_0}{\gamma_i}, & \epsilon_i > \frac{\psi_0}{(1-\delta)\gamma_i} \\ \frac{\psi_k}{\gamma_i}, & \frac{\psi_k}{(1-\delta)\gamma_i} \geq \epsilon_i > \frac{\psi_k}{(1+\delta)\gamma_i}, \quad 0 \leq k \leq l \\ 0, & \frac{\psi_l}{(1+\delta)\gamma_i} \geq \epsilon_i \geq 0 \\ r_i - \Upsilon(r_i - \epsilon_i), & \epsilon_i \leq 0 \end{cases} \quad (13)$$

Define $Y : \mathbb{R}^n \rightarrow \mathbb{R}^n$ as $Y(y) = (\Upsilon(y_1), \dots, \Upsilon(y_n))^\top$, and $\Gamma = \text{diag}(\gamma_i)$, then the closed loop system (11) can be rewritten to

$$J\dot{q} = f(K_p, K_v, B^\top q) - \Gamma(Y(y) - r) \quad (14)$$

since the identity in (12) is fulfilled.

The closed loop system in (14) has a discontinuous right hand side. Solutions to this system will here be considered in the sense of Krasovskii solutions.

Definition 4. (Bacciotti, 2004; Bacciotti and Ceragioli, 2006) A map $\varphi : I \rightarrow \mathbb{R}^n$ is a Krasovskii solution of an autonomous system of ordinary differential equations $\dot{x} = G(x)$, where $G : \mathbb{R}^n \rightarrow \mathbb{R}^n$, if it is absolutely continuous and for almost every $t \in I$ it satisfies the differential inclusion $\dot{\varphi}(t) \in KG(\varphi(t))$, where $KG(x) = \bigcap_{\delta>0} \overline{\text{co}} G(B_\delta(x))$ and $\overline{\text{co}} G$ is the convex closure of the set G .

Here, I is an interval of real numbers, possibly unbounded. If $G(x)$ is Lebesgue measurable and locally bounded, the operators K associates to $G(x)$ a set valued map which is upper semi-continuous, compact and convex valued. In particular, for each initial state x_0 there exists at least one Krasovskii solution of $\dot{x} = G(x)$ (Bacciotti and Ceragioli, 2006).

The Krasovskii solutions of (14) are absolutely continuous functions which satisfy the differential inclusion (Paden and Sastry, 1987)

$$J\dot{q} \in f(K_p, K_v, B^\top q) - \Gamma(K(Y(y)) - r), \quad (15)$$

where $K(Y(y)) \subseteq \times_{i=1}^n K(\Upsilon(y_i))$ and $K(\Upsilon(y_i))$ is given by

$$K(\Upsilon(y_i)) = r_i + \begin{cases} \frac{\psi_0}{\gamma_i}, & \epsilon_i > \frac{\psi_0}{(1-\delta)\gamma_i} \\ \frac{\psi_k}{\gamma_i}, & \frac{\psi_k}{(1-\delta)\gamma_i} > \epsilon_i > \frac{\psi_k}{(1+\delta)\gamma_i}, \\ & 0 \leq k \leq l \\ \Delta \frac{\psi_k}{\gamma_i}, & \epsilon_i = \frac{\psi_k}{(1+\delta)\gamma_i}, 0 \leq k \leq l \\ 0, & \frac{\psi_l}{(1+\delta)\gamma_i} > \epsilon_i \geq 0 \\ r_i - K(\Upsilon(r_i - \epsilon_i)), & \epsilon_i \leq 0 \end{cases} \quad (16)$$

for all $\Delta \in \{\frac{1-\lambda\delta}{1+\lambda\delta}, \lambda \in [0, 1]\}$.

4. STABILITY PROPERTIES OF CLOSED LOOP SYSTEM

In this section, the stability properties of the closed loop system introduced above will be examined. Subsequently, $f_K(\cdot)$ will be used to denote $f(K_p, K_v, \cdot)$. Furthermore, a more specific class of functions will be used in the expressions of $\mu(k_v, \cdot)$ and $\lambda(k_p, \cdot)$. This more specific class is motivated by the presence of turbulent² flows in the system (DePersis and Kalløe, 2009). The class of functions, which will be considered, is the following

$$\mu_i(k_{vi}, x_i) = k_{vi}|x_i| \quad (17)$$

$$\lambda_i(k_{pi}, x_i) = k_{pi}|x_i| \quad (18)$$

² Since the motivation for considering the new paradigm is reducing the diameters of the pipes used in the network, the likelihood for turbulent flows increases.

First, let the map $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$ be given as

$$F(z) = y(z) - \Gamma^{-1}f_K(B^\top z). \quad (19)$$

Proposition 5. (Jensen and Wisniewski, 2010) For the class of functions defined in (17) and (18), the map $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$ defined in (19) is a homeomorphism.

As a consequence of Proposition 5, there exists a unique vector $q^* \in \mathbb{R}^n$ for each vector of reference values $r \in \mathbb{R}^n$, and the relation between r and q^* is

$$r = y(q^*) - \Gamma^{-1}f_K(B^\top q^*). \quad (20)$$

This means that the expression for the closed loop system given in (14) can be replaced by

$$J\dot{q} \in \tilde{f}_K(\tilde{q}) - \Gamma(K(Y(y)) - y(q^*)) \quad (21)$$

where $\tilde{f}_K(\tilde{q}) = f_K(B^\top q) - f_K(B^\top q^*)$.

The following change of coordinates is made

$$\tilde{q} = q - q^*, \quad (22)$$

and the (Lyapunov) function $V : \mathbb{R}^n \rightarrow \mathbb{R}$ is defined as

$$V(\tilde{q}) = \frac{1}{2} \langle \tilde{q}, J\tilde{q} \rangle. \quad (23)$$

The time derivative of $V(\tilde{q})$ is then given as

$$\frac{d}{dt}V(\tilde{q}) = \langle \tilde{q}, J\dot{\tilde{q}} \rangle \quad (24)$$

$$\frac{d}{dt}V(\tilde{q}) \in \left\langle \tilde{q}, \tilde{f}_K(\tilde{q}) - \Gamma(K(Y(y)) - y(q^*)) \right\rangle \quad (25)$$

$$\frac{d}{dt}V(\tilde{q}) \in \left\langle \tilde{q}, \tilde{f}_K(\tilde{q}) \right\rangle - \langle \tilde{q}, \Gamma(K(Y(y)) - y(q^*)) \rangle \quad (26)$$

It can be shown that the following inequality holds (Jensen and Wisniewski, 2010)

$$w(\tilde{q}) \equiv \left\langle \tilde{q}, \tilde{f}_K(\tilde{q}) \right\rangle < 0, \quad \forall \tilde{q} \neq 0. \quad (27)$$

Now, the properties of the second term on the right hand side of (26) are examined. To that end, the parameter ψ_0 of the quantizer is first designed such that

$$r_i - \frac{\psi_0}{\gamma_i} \leq y_i(q_i^*) \leq r_i + \frac{\psi_0}{\gamma_i}, \quad i = 1, 2, \dots, n \quad (28)$$

Remark 6. Since the output functions are monotonic increasing and zero in $q_i = 0$, the following inequality holds:

$$(q_i - q_i^*)(y_i(q_i) - y_i(q_i^*)) > 0, \quad i = 1, 2, \dots, n. \quad (29)$$

Now, consider two different situations for $y_i(q_i^*)$ (the output of the system when $q = q^*$):

- 1) $y_i(q_i^*)$ is exactly equal to one of the quantization levels.

This is the case if the parameters γ_i, ψ_0, δ and l are designed such that $y_i(q_i^*) = r_i$ or such that there exist some $k \in \{0, 1, \dots, l\}$ so either $y_i(q_i^*) = r_i + \frac{\psi_k}{\gamma_i}$ if $y_i(q_i^*) > r_i$ or $y_i(q_i^*) = r_i - \frac{\psi_k}{\gamma_i}$ if $y_i(q_i^*) < r_i$.

- 2) $y_i(q_i^*)$ lies between two quantization levels.

This is the case if for $y_i(q_i^*) > r_i$, either $r_i < y_i(q_i^*) < r_i + \frac{\psi_l}{\gamma_i}$ or there exist some $k \in \{1, \dots, l\}$ such that $r_i + \frac{\psi_k}{\gamma_i} < y_i(q_i^*) < r_i + \frac{\psi_{k-1}}{\gamma_i}$. Or if for $y_i(q_i^*) < r_i$, either $r_i - \frac{\psi_l}{\gamma_i} < y_i(q_i^*) < r_i$ or there exist some $k \in \{1, \dots, l\}$ such that $r_i - \frac{\psi_{k-1}}{\gamma_i} < y_i(q_i^*) < r_i - \frac{\psi_k}{\gamma_i}$.

First, consider situation 1). In the range where $\Upsilon(y_i) = y_i(q_i^*)$, the following is fulfilled

$$(q_i - q_i^*)(\Upsilon(y_i) - y_i(q_i^*)) = 0 \quad (30)$$

and outside the above mentioned range

$$(q_i - q_i^*)(\Upsilon(y_i) - y_i(q_i^*)) > 0. \quad (31)$$

If situation 1) is fulfilled for every $i = 1, 2, \dots, n$, then

$$-\langle q - q^*, \Gamma(v - y(q^*)) \rangle \leq 0, \quad \forall v \in K(Y(y)), \quad (32)$$

since $\Gamma > 0$.

This shows $q = q^*$ is a globally asymptotically stable equilibrium point of the closed loop system, since

$$\frac{d}{dt}V(\tilde{q}) \leq w(\tilde{q}) < 0, \quad \forall q \neq q^* \quad (33)$$

where $\frac{d}{dt}V(\tilde{q})$ is given in (26) and $w(\tilde{q})$ is as defined in (27).

A more realistic situation is that there exist some $p \in \{1, 2, \dots, n\}$ (of course with a proper rearrangement of q) such that situation 2) is fulfilled for $q_1^*, q_2^*, \dots, q_p^*$.

Now, consider situation 2) for q_i^* . Denote the bounds in 2) α_i, β_i such that $\alpha_i < y_i(q_i^*) < \beta_i$. Whenever $y_i(q_i)$ is outside the range (α_i, β_i)

$$(q_i - q_i^*)(\Upsilon(y_i) - y_i(q_i^*)) > 0. \quad (34)$$

For a subset of the range (α_i, β_i) the sign of the product above changes.

Thus for the set $\mathcal{S} = \{q \in \mathbb{R}^n \mid y_i(q_i) \notin (\alpha_i, \beta_i), i = 1, \dots, p\}$, it can be guaranteed that $\frac{d}{dt}V(\tilde{q}) < w(\tilde{q}) < 0$.

Define $\mathcal{S}_1 = \mathbb{R}^n \setminus \mathcal{S}$. For a given point in the set \mathcal{S}_1 , there exists an index $s \leq p$ (with a proper rearrangement of q), such that

$$y_i(q_i) \in (\alpha_i, \beta_i), \quad i = 1, 2, \dots, s \quad (35)$$

Since $y_i(q_i)$ is proper, monotonically increasing and zero in $q_i = 0$, it admits a continuous inverse. Thus, the bound on $y_i(q_i)$ means that q_i is also bounded. Therefore, there exist some finite $m > 0$ such that

$$(q_i - q_i^*)(\Upsilon(y_i) - y_i(q_i^*)) > -m \quad (36)$$

and consequently, for each point $q \in \mathcal{S}_1$, there exist a finite $M > 0$ such that

$$\sum_{i=1}^s (q_i - q_i^*)(\Upsilon(y_i) - y_i(q_i^*)) > -M. \quad (37)$$

Let $M_{\mathcal{S}_1} > 0$ be the bound which fulfils

$$\sum_{i=1}^s (q_i - q_i^*)(\Upsilon(y_i) - y_i(q_i^*)) > -M_{\mathcal{S}_1}, \quad \forall q \in \mathcal{S}_1, \quad (38)$$

which exists, since $\alpha_i < y_i(q_i) < \beta_i$ for $i = 1, \dots, s$.

Let the set $\mathcal{S}_2 \subset \mathcal{S}_1$ denote the set for which the following holds

$$\sum_{i=s+1}^p (q_i - q_i^*)(\Upsilon(y_i) - y_i(q_i^*)) > M_{\mathcal{S}_1}. \quad (39)$$

Note that q_i^* is constant and $\Upsilon(y_i)$ is bounded, thus there exists finite q_i such that (39) is fulfilled, since q_i is unbounded for $i = s+1, \dots, p$.

Thus in the set \mathcal{S}_2 , the following inequality holds

$$-\langle q - q^*, \Gamma(v - y(q^*)) \rangle \leq 0, \quad \forall v \in K(Y(y)), \quad (40)$$

since $\Gamma > 0$.

Consequently $\frac{d}{dt}V(\tilde{q}) < w(\tilde{q}) < 0$ on the set \mathcal{S}_2 .

From the analysis above it is concluded that there exists some compact set $\mathcal{Q} \subset \mathbb{R}^n$, where $\mathcal{S}_1 \setminus \mathcal{S}_2 \subset \mathcal{Q}$, with the property that all trajectories of the system is attracted to \mathcal{Q} .

Furthermore, whenever the initial conditions of the closed loop system belong to a compact set, say \mathcal{Q} , it can be shown by applying Lyapunov arguments that practical output regulation of the system is achievable. That is: for any arbitrarily small positive number ε , and for any value of the quantization parameter $\delta \in (0, 1)$ there exist gains $\gamma_i^* > 0$ and parameters l and ψ_0 of the quantizer such that for all $\gamma_i > \gamma_i^*$, for any $r \in \mathcal{R}$, any Krasovskii solution $q(t)$ of the closed loop system with initial condition in \mathcal{Q} is attracted by the set $\{\epsilon \in \mathbb{R}^n \mid |\epsilon_i| \leq \varepsilon, i = 1, 2, \dots, n\}$, where $\epsilon_i = y_i - r_i$. The proof is similar to the one presented in (DePersis and Kallesøe, 2010) and is left out for brevity.

Since the result is global, the basin of attraction of the set \mathcal{Q} is the entire state space \mathbb{R}^n . Furthermore, since the result is independent on the number n of end-users, it will be possible to add or remove end-users in the system while maintaining stability in the sense that for the newly obtained system a compact set \mathcal{Q} which attracts the system trajectories will exist, given that (28) is fulfilled.

4.1 Quantization with Hysteresis

Using the quantizers defined in (13) may result in sliding modes arising along the switching surfaces, resulting in chattering and consequently the requirement for a large bandwidth. However, it is possible to replace the quantizer in (10) with an alternative for which it can be guaranteed that no sliding modes will arise (DePersis et al., 2010).

Due to space limitations no explicit proof of stability of the closed loop system using this alternate quantizer will be provided here. However, the proof can be done by a proper redefinition of the bounds α_i and β_i in the previous section.

5. NUMERICAL RESULTS

A numerical simulation of the system in Fig. 3 in closed loop with the proposed control has been performed, and the results are shown in Fig. 4. The proportional control actions defined in (9) and the quantizers including hysteresis has been used. A scenario, where the end-user connection consisting of $\{c_{12}, c_{13}, c_{14}\}$, has been removed from and later re-inserted into the system has been simulated. The Figure shows that the end-user connection is removed at time 300 s and re-inserted again at time 600 s. The parameters used in the simulation are; $\gamma_1 = \gamma_2 = 2$, $\delta = 0.5$, $\psi_0 = 1$ and $l = 2$. The reference values are $r_1 = r_2 = 0.5$ Bar, which is indicated by the solid line in the middle two plots in Fig. 4. Contrary to the result with the continuous proportional control actions (Jensen and Wisniewski, 2010), it is evident from Fig. 4 that a single unique equilibrium point can generally not be achieved when the quantized version of the proportional control actions are used. For instance a limit cycle-type behaviour is achieved for the single end-user system between time 300 s and 600 s.

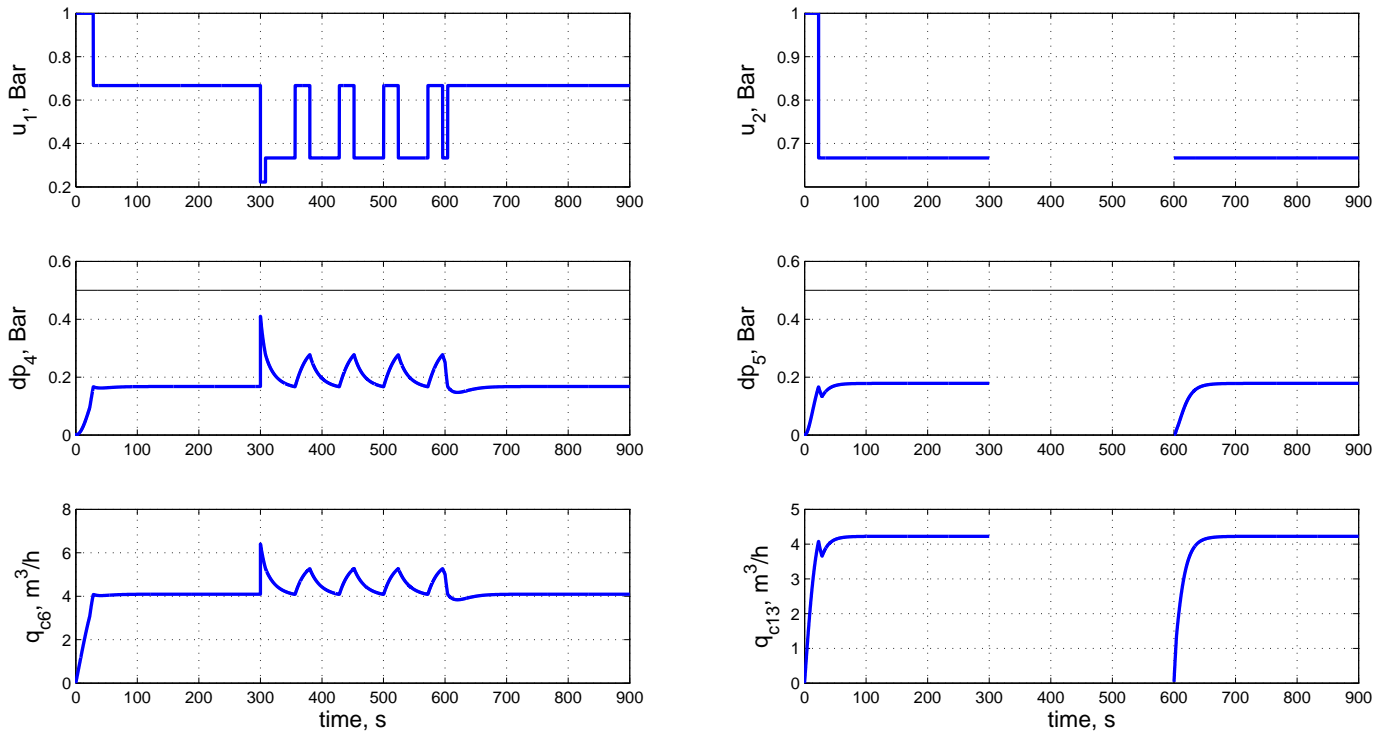


Fig. 4. Result of a numerical simulation of the two end-user system in Fig. 3. The figure shows control inputs u_1 and u_2 , the controlled variable dp_4 and dp_5 , and the flow through valve c_6 and c_{13} obtained with the proportional feedback control. At time 300 s, the end-user connection consisting of $\{c_{12}, c_{13}, c_{14}\}$ is removed from the system. At time 600 s the end-user connection is re-inserted into the system.

6. CONCLUSION

An industrial case study involving a large-scale hydraulic network underlying a district heating system was investigated. The results show that the closed loop system using a set of quantized proportional feedback control actions is globally stable in the sense that there exists a compact set \mathcal{Q} which attracts all system trajectories. Furthermore, it has been shown that this set can be made arbitrarily small by choosing a proper set of parameters for the feedback controller and quantizer. Specifically, for the result to hold, the bounds in (28) has to be fulfilled. Since the result is global and independent on the number of end-users in the system, a set \mathcal{Q} with the above mentioned properties will also exist for the newly obtained system if it should be necessary to add or remove end-users to/from the system. This, along with the decentralized nature of the control structure, will make it easy to implement structural changes in the system, while maintaining closed loop stability.

Future extensions of the results presented in this paper, will consist of an investigation of quantized proportional controllers, which are constrained to deliver only positive control signals. This is important since the (centrifugal) pumps used in the network are only able to deliver positive pressure inputs to the system. Furthermore, it will be interesting to investigate closed loop stability using proportional-integral control actions in order to accommodate for the output regulation error, which is present with the proportional control actions.

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